



A generalized Milne–Thomson theorem for the case of parabolic inclusion

Yu.V. Obnosov *

Department of Mechanics and Mathematics, Kazan State University, University Street, 17, 420008 Kazan, Tatarstan, Russia

ARTICLE INFO

Article history:

Received 27 June 2007

Received in revised form 4 April 2008

Accepted 1 May 2008

Available online 10 May 2008

Keywords:

Heterogeneous media

Circular theorem

Analytic functions

ABSTRACT

Complex analysis methods are applied to determine a velocity field of seepage in a heterogeneous infinite planar medium consisting of two dissimilar homogeneous components with a parabolic interface. New cases with arbitrary singularities of the principal part of a required complex potential are considered.

© 2008 Elsevier Inc. All rights reserved.

1. Introduction

We consider the problem of determination of a seepage field in a planar porous medium with a parabolic division boundary between two homogeneous components having the hydraulic conductivity k_1 and k_2 . The interface separates the inclusion S_2 from the ambient medium S_1 as is shown in Fig. 1. We assume a steady-state, fully-saturated, Darcian flow in a layer (e.g., a confined aquifer) of a constant thickness. Then the Darcian velocity $\mathbf{v}_p(x, y) = (v_{px}, v_{py})$, $(x, y) \in S_p$, is potential and solenoidal in each isotropic component S_p , $p = 1, 2$ [1]:

$$\operatorname{div} \mathbf{v}_p(x, y) = 0, \quad \operatorname{rot} \mathbf{v}_p(x, y) = 0. \quad (1.1)$$

Along the parabolic interface $\mathcal{L} = \bar{S}_1 \cap \bar{S}_2$ the limit values of the vectors \mathbf{v}_p and \mathbf{v}_p/k_p satisfy the following refraction conditions:

$$[\mathbf{v}_1(x, y)]_n = [\mathbf{v}_2(x, y)]_n, \quad k_2[\mathbf{v}_1(x, y)]_\tau = k_1[\mathbf{v}_2(x, y)]_\tau, \quad (x, y) \in \mathcal{L}. \quad (1.2)$$

Here the subscripts n and τ denote the normal and tangential components of the vectors. Problem (1.1), (1.2) can be equivalently formulated in terms of the complex potential $\mathbf{w}_p(x, y) = (\varphi_p(x, y), \psi_p(x, y))$, where $\varphi_p(x, y)$ is the velocity potential and $\psi_p(x, y)$ is the stream function within the component S_p , and $\partial\varphi_p/\partial x = v_{px}$, $\partial\varphi_p/\partial y = v_{py}$.

It is well-known that an explicit analytical solution of the above stated problem can be obtained for few heterogeneities only. If we exclude the trivial case of perfectly aligned strata [1], then the simplest two-component composite consists of a circular inclusion. Solution of the corresponding boundary value problem constitutes the famous Milne–Thomson circle theorem [2]. In [2]. In [3,24] the Milne–Thomson circle theorem was generalized for the case when a required complex potential had a finite number of singularities arbitrary situated on the plane. These singularities physically represent pumping and/or injection wells (sinks/sources, [4,5]), river-locks or dams (vortexes, [6]) and immersed obstacles (dipoles, [7]).

* Tel.: +7 8432 315 278; fax: +7 8432 382 209.

E-mail address: Yurii.Obnosov@ksu.ru